OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Express $\dfrac{1}{(2x+1)(x^2+1)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $\sqrt[3]{1+3x}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[5]**

3 Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2\sin\theta - 3\cos\theta$. **[6]**

4 A curve has parametric equations

$$x = 2\sin\theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter $\theta = \frac{1}{3}\pi$. **[5]**

(ii) Find y in terms of x. **[2]**

5 Solve the equation $\operatorname{cosec}^2\theta = 1 + 2\cot\theta$, for $-180° \leqslant \theta \leqslant 180°$. **[6]**

6 Fig. 6 shows the region enclosed by part of the curve $y = 2x^2$, the straight line $x + y = 3$, and the y-axis. The curve and the straight line meet at P (1, 2).

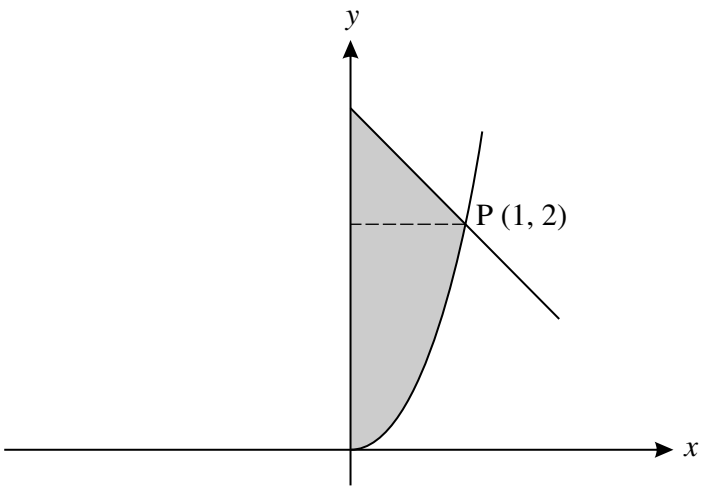

Fig. 6

The shaded region is rotated through 360° about the y-axis. Find, in terms of π, the volume of the solid of revolution formed. **[7]**

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.]

Section B (36 marks)

7 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is k metres.

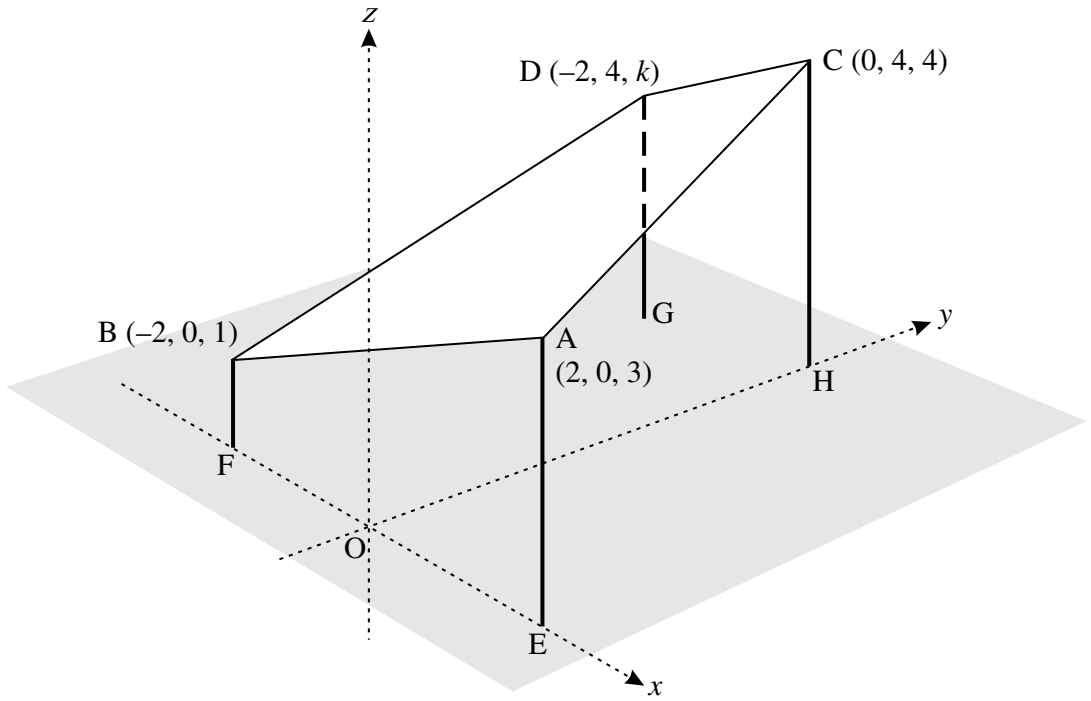

Fig. 7

(i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence calculate the angle BAC. **[6]**

(ii) Verify that the equation of the plane ABC is $x + y - 2z + d = 0$, where d is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. **[7]**

(iii) Given that A, B, D and C are coplanar, show that $k = 3$.

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. **[5]**

[Question 8 is printed overleaf.]

8 Water is leaking from a container. After t seconds, the depth of water in the container is x cm, and the volume of water is V cm^3, where $V = \frac{1}{3}x^3$. The rate at which water is lost is proportional to x, so that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -kx$, where k is a constant.

(i) Show that $x\dfrac{\mathrm{d}x}{\mathrm{d}t} = -k$. **[3]**

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that $x = \sqrt{100 - 2kt}$. **[4]**

(iii) Given that the container empties after 50 seconds, find k. **[2]**

Once the container is empty, water is poured into it at a constant rate of 1 cm^3 per second. The container continues to lose water as before.

(iv) Show that, t seconds after starting to pour the water in, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1-x}{x^2}$. **[2]**

(v) Show that $\dfrac{1}{1-x} - x - 1 = \dfrac{x^2}{1-x}$.

Hence solve the differential equation in part **(iv)** to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \tfrac{1}{2}x^2 - x.$$ **[6]**

(vi) Show that the depth cannot reach 1 cm. **[1]**

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Morning

Duration: 1 hour 30 minutes

* 4 7 5 4 A *

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1

2

3	

4 (i)

4 (ii)

5

6

Section B (36 marks)

7 (i)

7 (i) **(continued)**

7 (ii)

(answer space continued overleaf)

7 (ii)	**(continued)**
7 (iii)	

7 (iii)	(continued)

8 (i)

8 (ii)

8 (iii)

8 (iv)

8 (v)

8 (vi)	

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1 $\frac{1}{(2x+1)(x^2+1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+1}$ $\Rightarrow \quad 1 = A(x^2+1) + (Bx+C)(2x+1)$ $x = -½ : 1 = 1¼ A \Rightarrow A = 4/5$ coeff of x^2: $0 = A + 2B \Rightarrow B = -2/5$ constants: $1 = A + C \Rightarrow C = 1/5$	M1 M1 B1 B1 B1 [5]	correct form of partial fractions mult up and equating or substituting oe soi www www www	for omission of B or C on numerator, M0, M1, then (x= -1/2, A= 4/5) B1, B0, B0 is possible. for $\frac{A+Dx}{2x+1} + \frac{Bx+C}{x^2+1}$, M1,M1 then B1 for both A=4/5 and D=0, B1, B1 is possible. isw for incorrect assembly of final partial fractions following correct A,B & C. condone omission of brackets for second M1 only if the brackets are implied by subsequent working.
2 $(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2 + \ldots$ $= 1 + x - x^2 + \ldots$ Valid for $-1 \le 3x \le 1$ $\Rightarrow \quad -1/3 \le x \le 1/3$	M1 A1 A1 M1 A1 [5]	correct binomial coefficients $1 + x \ldots$ $\ldots - x^2$ or $\lvert 3x \rvert \le 1$ oe or $\lvert x \rvert \le 1/3$ (correct final answer scores M1A1)	ie 1, 1/3, (1/3)(-2/3)/2 not nCr form simplified www in this part simplified www in this part, ignore subsequent terms using $(3x)^2$ as $3x^2$ can score M1B1B0 condone omission of brackets if $3x^2$ is used as $9x^2$ do not allow MR for power 3 or -1/3 or similar **condone inequality signs throughout** or say < at one end and ≤ at the other condone $-1/3 \le \lvert x \rvert \le 1/3$, $x \le 1/3$ is M0A0 the last two marks are not dependent on the first three
3 $2\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2, \; R\sin\alpha = 3$ $\Rightarrow R^2 = 2^2 + 3^2 = 13, R = \sqrt{13}$ $\tan\alpha = 3/2$, $\Rightarrow \quad \alpha = 0.983$ minimum $1 - \sqrt{13}$, maximum $1 + \sqrt{13}$	 M1 B1 M1 A1 B1 B1 [6]	 correct pairs $R = \sqrt{13}$ or 3.61 or better 0.98 or better or –2.61, 4.61 or better	 condone wrong sign at this stage correct division, ft from first M1 radians only accept multiples of π that round to 0.98 allow B1, B1ft for $1-\sqrt{R}$ and $1+\sqrt{R}$ for their R to 2dp or better

4(i) $x = 2\sin\theta$, $y = \cos 2\theta$ When $\theta = \pi/3$, $x = 2\sin\pi/3 = \sqrt{3}$ $y = \cos 2\pi/3 = -½$	B1 B1	$x = \sqrt{3}$ $y = -½$	exact only (isw all dec answers following exact ans)
EITHER $dx/d\theta = 2\cos\theta$, $dy/d\theta = -2\sin 2\theta$	M1	$dy/dx = (dy/d\theta) / (dx/d\theta)$ used	ft their derivatives if right way up (condone one further minor slip if intention clear) condone poor notation
$\Rightarrow \quad \frac{dy}{dx} = \frac{-\sin 2\theta}{\cos\theta}$	A1	any correct equivalent form	can isw if incorrect simplification
$= \frac{-\sin 2\pi/3}{\cos\pi/3} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$	A1	exact www	
……....	……..	……	
OR expressing y in terms of x, $y=1-x^2/2$ $\frac{dy}{dx} = -x$ or $-2\sin\theta$ $= -\sqrt{3}$	M1 A1 A1	 exact www	
	[5]		
(ii) $y = 1 - 2\sin^2\theta = 1 - 2(x/2)^2 = 1 - ½x^2$	M1A1 [2]	or reference to (i) if used there	for M1, need correct trig identity and attempt to substitute for x allow SC B1 for $y=\cos 2\arcsin(x/2)$ or equivalent

5 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow$ $1 + \cot^2\theta = 1 + 2\cot\theta$ $\Rightarrow$ $\cot^2\theta - 2\cot\theta = 0$ $\Rightarrow$ $\cot\theta(\cot\theta - 2) = 0$ $\Rightarrow$ $\cot\theta = 0$, **and** $\cot\theta = 2$, $\tan\theta = ½$ $\Rightarrow$ $\theta = 26.6°, -153.4°, -90°, 90°$ ……. **OR** $\frac{1}{\sin^2\theta} = 1 + \frac{2\cos\theta}{\sin\theta} = \frac{\sin\theta + 2\cos\theta}{\sin\theta}$ $\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta - 1 = 0$ $\Rightarrow 2\sin\theta\cos\theta - \cos^2\theta = 0$ $\Rightarrow \cos\theta(2\sin\theta - \cos\theta) = 0$ $\Rightarrow \cos\theta = 0$, **and** $\tan\theta = ½$ $\theta = 26.6°, -153.4°, -90°, 90°$	M1 M1 M1 B3,2,1,0 M1 M1 M1 B3,2,1,0 [6]	correct trig identity used factorising oe **both** needed and $\cot\theta = 1/\tan\theta$ soi $-90°, 90°, 27°, -153°$ or better www …… correct trig equivalents and a one line equation (or common denominator) formed use of Pythagoras and factorising **both** needed and $\tan\theta = \sin\theta/\cos\theta$ oe soi accept 27º, -153º as above ……. answers, no working, award B3,2,1,0 (it is possible to score say M1 then B3 ow)	(use of $1-\cot^2\theta$ could lead to M0 M1 M1 B1) allow if $\cot\theta = 0$ not seen (ie quadratic equation followed by $\cot\theta - 2 = 0$ or $\cot\theta = 2$) (omission of $\cot\theta = 0$ could gain M1, M1, M0, B1) ……... as above allow if $\cos\theta = 0$ not seen (as above) …… **in both cases**, -1 if **extra** solutions in the range are given (dependent on at least B1 being scored)-not their incorrect solutions eg 26.6°,-153.4°, 0°,180°,-180° would obtain B1 -1 MR if answers given in radians (-π/2,π/2,0.464, -2.68 (-1.57.1.57) or multiples of π that round to these, or better) (dependent on at least B1 being scored) to lose both of these, at least B2 would need to be scored.
6 Vol = vol of rev of curve + vol of rev of line vol of rev of curve = $\int_0^2 \pi x^2 \, dy$ $= \int_0^2 \pi \frac{y}{2} dy$ $= \pi\left[\frac{y^2}{4}\right]_0^2$ $= \pi$ height of cone = 3 – 2 = 1 so vol of cone = 1/3 π 1^2 x1 = π/3 so total vol = 4π/3	M1 M1 B1 A1 B1 B1 A1 [7]	(soi) at any stage substituting $x^2 = y/2$ $\left[\frac{y^2}{4}\right]$ h=1 soi www cao	 for M1 need π, substitution for x^2, (dy soi), intention to integrate and correct limits even if π missing or limits incorrect or missing cao OR $\pi\int_2^3 (3-y)^2 dy$ M1(even if expanded incorrectly) =π/3 A1 www

Section B

7(i)	$\overrightarrow{AB} = \begin{pmatrix}-4\\0\\-2\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-2\\4\\1\end{pmatrix}$	B1B1		condone rows
	$\cos BAC = \dfrac{\begin{pmatrix}-4\\0\\-2\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix}}{AB.AC} = \dfrac{(-4).(-2)+0.4+(-2).1}{\sqrt{20}\sqrt{21}}$	M1 M1	dot product evaluated **cos** BAC= dot product / |AB|. |AC|	**substituted,** ft their vectors AB, AC for method only need to see method for modulae as far as √…
	$= 0.293$	A1	0.293 or cos ABC=correct numerical expression as RHS above, or better	use of vectors BA and CA could obtain B0 B0 M1 M1 A1 A1
⇒	BAC = 73.0°	A1 [6]	or rounds to 73.0º (accept 73º www)	(or 1.27 radians)
(ii)	A: $x + y - 2z + d = 2 - 6 + d = 0$			alternatively, finding the equation of the plane using any valid method (eg from vector equation, M1 A1 for using valid equation and eliminating both parameters, A1 for required form, or using vector cross product to get $x+y-2z=c$ oe M1 A1,finding c and required form, A1, or showing that two vectors in the plane are perpendicular to normal vector M1 A1 and finding d, A1) oe
⇒	$d = 4$	M1	substituting one point	
	B: $-2 + 0 - 2\times1 + 4 = 0$	DM1	evaluating for other two points	
	C: $0 + 4 - 2\times4 + 4 = 0$	A1	$d = 4$ www	
	Normal $\mathbf{n} = \begin{pmatrix}1\\1\\-2\end{pmatrix}$	B1	stated or used as normal anywhere in part (ii)	
	$\mathbf{n}.\begin{pmatrix}0\\0\\1\end{pmatrix} = \dfrac{-2}{\sqrt{6}} = \cos\theta$	M1 A1	finding angle between normal vector and **k** allow ±2/√6 or 144.7º for A1	(may have deliberately made +ve to find acute angle)
⇒	$\theta = 144.7°$			do not need to find 144.7º explicitly
⇒	acute angle = 35.3°	A1 [7]	or rounds to 35.3º	(or 0.615 radians)
(iii)	At D, $-2 + 4 - 2k + 4 = 0$	M1	substituting into plane equation	
⇒	$2k = 6$, $k = 3$ *	A1	**AG**	
	$\overrightarrow{CD} = \begin{pmatrix}-2\\0\\-1\end{pmatrix} = \dfrac{1}{2}\overrightarrow{AB}$	M1	$\overrightarrow{CD} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$	finding vector CD (or vector DC)
⇒	CD is parallel to AB	A1		or DC parallel to AB or BA oe (or hence two parallel sides, if clear which) but A0 if their vector CD is vector DC
	CD : AB = 1 : 2	B1 [5]	mark final answer **www** allow CD:AB=1/2, √5:√20 oe, AB is twice CD oe	for B1 allow vector CD used as vector DC

8(i) $\frac{dV}{dt} = -kx$ $V = 1/3\ x^3 \Rightarrow dV/dx = x^2$ $\frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt} = x^2\frac{dx}{dt}$ $\Rightarrow \quad x^2\frac{dx}{dt} = -kx$ $\Rightarrow \quad x\frac{dx}{dt} = -k$ *	B1 M1 A1 [3]	oe eg $dx/dt=dx/dV\ .\ dV/dt = 1/x^2.\ -kx = -k/x$ **AG**	
(ii) $x\frac{dx}{dt} = -k \quad \Rightarrow \quad \int x\,dx = \int -k\,dt$ $\Rightarrow \quad ½\,x^2 = -kt + c$ When $t = 0, x = 10 \Rightarrow 50 = c$ $\Rightarrow \quad ½\,x^2 = 50 - kt$ $\Rightarrow \quad x = \sqrt{(100 - 2kt)}$ *	M1 A1 B1 A1 [4]	separating variables and intention to integrate condone absence of c finding c correctly ft their integral of form $ax^2 = bt+c$ where a,b non zero constants **AG**	
(iii) When $t = 50, x = 0$ $\Rightarrow \quad 0 = 100 - 100\,k \Rightarrow k = 1$	M1 A1 [2]		
(iv) $dV/dt = 1 - kx = 1 - x$ $\Rightarrow \quad x^2 dx/dt = 1 - x$ $\Rightarrow \quad \frac{dx}{dt} = \frac{1-x}{x^2}$ *	M1 A1 [2]	for $dV/dt = 1-kx$ or better **AG**	
(v) $\frac{1}{1-x} - x - 1 = \frac{1-(1-x)x-(1-x)}{1-x}$ $= \frac{1-x+x^2-1+x}{1-x} = \frac{x^2}{1-x}$ * $\int \frac{x^2}{1-x}dx = \int dt \quad \Rightarrow \int (\frac{1}{1-x} - x - 1)dx = t + c$ $\Rightarrow \quad -\ln(1-x) - ½\,x^2 - x = t + c$ When $t = 0, x = 0 \Rightarrow c = -\ln 1 - 0 - 0 = 0$ $\Rightarrow \quad t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x$ *	M1 A1 M1 A1 B1 A1 [6]	combining to single fraction **AG** separating variables & subst for $x^2/(1-x)$ and intending to integrate condone absence of c finding c for equation of correct form eg $c = 0$, or $\pm \ln 1$ (allow c=0 without evaluation here) cao **AG**	or long division or cross multiplying check signs need both sides of integral accept $\ln(1/(1-x))$ as $-\ln(1-x)$ www ie $a\ln(1-x)+bx^2+dx=et+c$ a,b,d,e non zero constants do not allow if c=0 without evaluation
(vi) understanding that ln (1/0) or 1/0 is undefined oe	B1 [1]	www	ln (1/0) = ln 0, 1/0 = ∝ and ln (1/0) = ∝ are all B0

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Monday 13 June 2011
Morning

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

1 In lines 59 and 60, the text says "In that case the proportion suffering such an attack would be 6.4%."

Explain how this figure was obtained. **[1]**

1	

2 **(i)** In lines 8 to 10, the article says "Some banks do not allow numbers that begin with zero, numbers in which the digits are all the same (such as 5555) or numbers in which the digits are consecutive (such as 2345 or 8765)."

How many different 4-digit PINs can be made when all these rules are applied? **[3]**

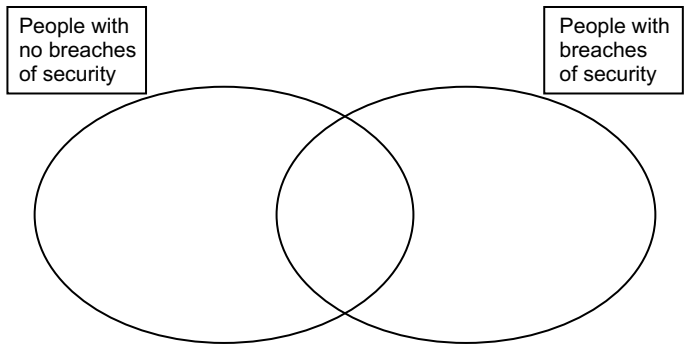
(ii) At the time of writing, the world population is 6.7×10^9 people. Assuming that, on average, each person has one card with a 4-digit PIN (subject to the rules in part **(i)** of this question), estimate the average number of people holding cards with any given PIN. Give your answer to an appropriate degree of accuracy. **[2]**

2 (i)	
2 (ii)	

3 In lines 46 and 47, the text says "Of the 11 people with unauthorised transactions, 3 could explain them as breaches of card security (typically losing the card) but 9 could not … ."

Place numbers in the three regions of the diagram consistent with the information in this sentence. **[2]**

3



4 In lines 101 and 102, the text says "The total number of transactions for those who responded has been estimated as 100 000 for the $3\frac{1}{2}$ years covered by the survey."

Estimate the number of transactions per person per day that would give this figure. **[2]**

4

5 The survey described in the article was based on a small sample.

State one conclusion which is unlikely to be influenced by the size of the sample. **[1]**

5

6 A bank has detection software that can be set at two different levels, 'Mild' and 'Severe'.

- When it is set at Mild, 0.1% of all transactions are queried.
- When it is set at Severe 0.5% of all transactions are queried.

(i) One day the bank has 500 000 transactions.

The software is set on 'Mild'. There are 480 false positives. Only $\frac{1}{3}$ of the unauthorised transactions are queried. Complete the table. **[3]**

(ii) What is the ratio of false positives to false negatives? **[1]**

(iii) If the software had been set on 'Severe' for the same set of 500 000 transactions, with the total numbers of authorised and unauthorised transactions the same as in part **(i)** of this question, the number of false negatives would have been 5. What would the ratio of false positives to false negatives have been with this setting? **[3]**

6 (i)

Transactions	Authorised	Unauthorised	Total
Queried			
Not queried			
Total			500 000

6 (ii)

6 (iii)

[A copy of the table is provided below for your working.]

Transactions	Authorised	Unauthorised	Total
Queried			
Not queried		5	
Total			500 000

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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Monday 13 June 2011
Morning

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

- This insert contains the text for use with the questions.
- This document consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this insert for marking; it should be retained in the centre or destroyed.

Card safety

A court case

In a recent (2009) court case, a man claimed that his bank owed him £2100. The money had been taken from his bank account in eight withdrawals from two cash machines, apparently using his bank card, but he said that he had neither made these withdrawals himself nor asked anyone else to make them. His card used chip and PIN technology. The bank had refused to refund his money.

The letters PIN stand for Personal Identification Number and refer to a 4-digit number that is needed to authorise transactions using the card, such as withdrawing money. Some banks do not allow numbers that begin with zero, numbers in which the digits are all the same (such as 5555) or numbers in which the digits are consecutive (such as 2345 or 8765).

When a bank issues you with a card, various security conditions come with it.

- You are not allowed to tell anyone the PIN.
- You are not allowed to write the PIN down; you must remember it.
- You may not lend the card to anyone else.
- If you lose the card, you must report the loss to the bank.

In the 2009 court case, there was no dispute that the withdrawals had taken place. The question was which of the following possible explanations was correct.

1. The man was making a dishonest claim. If so, the man is clearly at fault.
2. The bank had made an error, for example by taking money from this man's account by mistake, and so is at fault.
3. A breach of the card's security conditions, as stated above, had allowed a thief to withdraw the money. If so, the man is legally at fault.
4. A thief had been able to withdraw the money without a breach of the card's security conditions. If so, the fault lies with the bank's systems.

Two of these explanations (1 and 3) say that the man is at fault; the other two (2 and 4) put the blame on the bank.

Explanations 3 and 4 involve successful 'attacks' by a thief.

The judge decided in favour of the bank. His written judgment caused some people concern. Part of it could be read as meaning that chip and PIN technology is absolutely secure; if so, it would imply that it is impossible for a thief to copy, or 'clone', someone else's card or to break a bank's security.

A survey of card users

Following this judgment, MEI conducted a small survey to provide information about the situation, and particularly the four possible explanations. Those taking part were mathematics teachers attending a conference in June 2009; 250 questionnaires were given out and 80 returned. The survey was based on people's experience with their banks.

Banks use software designed to detect suspicious transactions; if one is detected, the bank usually contacts the card-holder to check whether the transaction should go ahead. The survey asked people whether they had been contacted by their banks about a suspicious transaction.

If the answer was Yes, they were then asked to answer a further question as to whether they had

authorised the transaction. An answer of Yes to this further question meant that the transaction was genuine, and an answer of No that an attack had been detected.

- 46 of the 80 respondents had been contacted by their banks, many of them several times.
- Most of the transactions had in fact been authorised, but 11 of the 46 people had been contacted about unauthorised transactions.
- Of the 11 people with unauthorised transactions, 3 could explain them as breaches of card security (typically losing the card) but 9 could not (one person was in both categories).

The survey then went on to ask about cases that had not been picked up by the banks' detection software, resulting in unauthorised withdrawals from people's accounts.

- 21 people reported unauthorised withdrawals.
- Of these, 9 people could explain them as breaches of card security and 13 could not (again one person was in both categories).

In total, 16 out of the 80 people who responded to the survey had been the subject of attacks in which there was no breach of the card's security conditions. Some of the attacks had been stopped by their banks but others had resulted in money being withdrawn from their accounts; some people reported both of these. If the survey results are reasonably representative, they would suggest that, in the course of the $3\frac{1}{2}$ years covered by the survey, 20% of people had suffered an attack without any breach of their cards' security. This may be an overestimate. Only 80 out of 250 people returned the questionnaire; maybe all the 170 who did not return it had nothing to report. In that case the proportion suffering such an attack would be 6.4%.

The conclusion that attacks can happen without breaches of card security is supported by the fact that banks are prepared to bear the considerable costs that must be involved in the process of carrying out checks.

Possible explanations

In a typical court case involving card security, the claimant has had money withdrawn from an account and the bank has refused to refund it. There is no dispute that the withdrawal has taken place. The four possible explanations on the first page apply and the results from the survey make it possible to say something about them.

In the first explanation, it is the claimant who has withdrawn the money and is then saying it was someone else. The survey suggests that the other explanations are also possible. Whether the court judges the claimant to be telling the truth must depend on other evidence.

The next possible explanation is that the bank made an error, and this can happen. One of the responses to the questionnaire said

> *"We went to the bank and spoke at length with the manager. We were fully reimbursed and had a grovelling apology."*

That leaves the two explanations that involve the money being taken by a thief, with or without a breach of the card's security. The survey also identified the number of transactions, as well as the number of people, subject to attacks. There were a total of 42 attacks; several people reported more than one attack. 13 of the attacks could be explained by breaches of card security and 29 could not.

So the data would suggest that, if there has been an attack, the probabilities of the two explanations of breach and no breach of card security are $\frac{13}{42}$ and $\frac{29}{42}$. These figures are based on a small sample and so it would be better to think of them as about $\frac{1}{3}$ and $\frac{2}{3}$. In civil cases, courts decide the outcome on a 'balance of probabilities'. The probabilities of $\frac{1}{3}$ and $\frac{2}{3}$ are so close together that a court would

be unwilling to decide the matter on the basis of them alone, and would look for other evidence before reaching a decision.

The banks

The survey went on to ask those who reported unauthorised withdrawals what happened next. In nearly all cases the bank had refunded the money but in one case this had not happened.

One of the responses to the questionnaire said

> *"The bank described two transactions in the space of 3 or 4 hours. One for about £40 in a shop in London and the other for over £500 at an expensive restaurant/club in London. I was in Paris at the time of these transactions. The bank refunded both amounts after I filled in a form. … I assume that someone had managed to clone my card somehow."*

Clearly fraud can cost the banks a lot of money.

If a bank refuses to pay, the next course of action open to someone who has lost money in this way is to contact the independent Financial Ombudsman. A few of these cases are then taken further and end up in court.

Detecting fraud

The survey provided information about the banks' success in detecting unauthorised transactions. The total number of transactions for those who responded has been estimated as 100 000 for the $3\frac{1}{2}$ years covered by the survey. Table 1 shows data from the survey and, in brackets, figures derived from the estimate of 100 000 transactions.

Transactions	Authorised	Unauthorised	Total
Queried	139	19	158
Not queried	(99 819)	23	(99 842)
Total	(99 958)	42	(100 000)

Table 1

Table 1 illustrates the problems faced by the banks. They check a very large number of transactions, query quite a small proportion of them and succeed in stopping a small number of unauthorised transactions. However, despite all this effort, the figures in Table 1 suggest that they only catch about half of the attacks.

The entries in a table like this are often described using the terms in Table 2.

Transactions	Authorised	Unauthorised
Queried	False positives	True positives
Not queried	True negatives	False negatives

Table 2

A ‘positive’ is a transaction that is identified by a bank’s computer software as suspect and so is queried. The identification is ‘false’ if the transaction was in fact authorised and it is ‘true’ if the transaction was unauthorised.

Similarly, a ‘negative’ is a transaction that is not identified as suspect and this non-identification may be true or false.

So, if the software gives a warning when there is no attack, a false positive results; 139 of these are recorded in Table 1 and, apart from some inconvenience, they are quite harmless. If, however, the software fails to give a warning when there really is an attack, a false negative occurs, resulting in unauthorised withdrawals and these are the serious cases; there are 23 of them in Table 1.

The number of false negatives can be reduced by making the warning criteria in the software more severe, but the effect will inevitably be that the number of false positives rises: the more severe warning criteria will pick out more authorised transactions. Thus the fewer the false negatives, the greater the number of false positives, and vice-versa.

The detection software may be thought of as the front line in the ongoing struggle between thieves and banks. Once thieves learn how it is programmed, they can find ways to defeat it. Consequently the information is considered to be top secret by the banks.

Finally, a piece of advice. Never let anyone else use your card. Legally, your PIN is an electronic signature and so allowing someone else to use it is the equivalent of telling them to forge your signature.

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Mathematics (MEI)

Advanced GCE

Unit **4754B:** Applications of Advanced Mathematics: Paper B

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

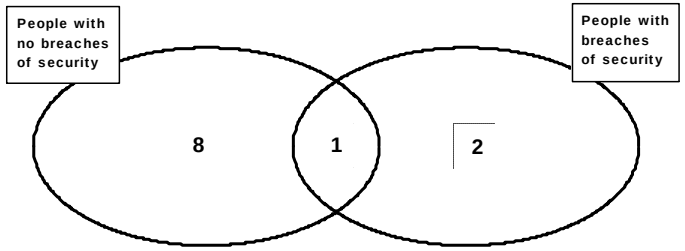
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\frac{16}{250} = 6.4\%$ * or $\frac{16}{250}$ x 100 = 6.4*	B1 [1]	or $\frac{250-(64+170)}{250}$ =6.4% oe	need evaluation
2	(i)		The smallest possible PIN that does not begin with zero is 1000 and the largest is 9999, giving 9000. However the 9 numbers 1111, 2222, … 9999 are disallowed. The other disallowed numbers are 1234, 2345, … 6789 (6 numbers) And 9876, 8765, … 3210 (7 numbers). So, in all, there are 9000 – (9 + 6 + 7) = 8978 possible PINs	M1 M1 A1 [3]	from a correct starting point (eg 10,000 or 9000), clear attempt to eliminate (or not include) numbers starting with 0 clear attempt to eliminate **all** **three** of these categories (with approx correct values in each category) **if unclear, M0** M marks not dependent SC 8978 www B3	Alt1) for M1 (no 0 start), nos starting with 1,2,7,8,9 give 1000-2, nos starting with 3,4,5,6 give 1000-3 =5(1000-2)+4(1000-3)=8978 M1,A1 or2) eg starting with1, 1,not2,any,any+1,2,not3,any +1,2,3,not4 =900+90+9=999-(1111term)=998 can lead to 5(900+90+9-1)+4(900+90+9-2)=8978 oe
2	(ii)		$\frac{6\ 700\ 000\ 000}{8978} = 746\ 269$ The average is about 750 000.	M1 A1 [2]	ft from (i) ft	 accept 2sf (or 1sf) only for A1
3				M1 A1 [2]	numbers total 11 all correct	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td></td><td></td><td>100 000 transactions from 80 people over 3½ years with 365 days per year
$\frac{100\ 000}{(80\times3.5\times365)}(=0.978...)$
Approximately 1 transaction per person per day</td><td>M1
A1
[2]</td><td>cao</td><td>allow approximate number of days in a year eg 360 for M1 A1</td></tr>
<tr><td>5</td><td></td><td></td><td>Allow any one of the following for 1 mark
An attack can happen without a breach of the card's security.
The probabilities that a successful attack followed or did not follow a breach of card security are so close that a court would look for other evidence before reaching a decision.
In many cases of unauthorised withdrawals the banks refund the money.
The banks' software does not detect all the attacks that occur.</td><td>B1
[1]</td><td>only accept versions of these statements</td><td></td></tr>
<tr><td>6</td><td>(i)</td><td></td><td>
<table>
<tr><th>Transactions</th><th>Authorised</th><th>Un-authorised</th><th>Total</th></tr>
<tr><td>Queried</td><td>480</td><td>20</td><td>500</td></tr>
<tr><td>Not queried</td><td>499 460</td><td>40</td><td>499 500</td></tr>
<tr><td>Total</td><td>499 940</td><td>60</td><td>500 000</td></tr>
</table>
</td><td>B1
B2
[3]</td><td>for top row 480, 20, 500
all five other entries correct</td><td>(500 000 is given)
allow B1 for three or four correct from 499460,40,499500,499940,60</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th><th></th></tr>
<tr><td>6</td><td>(ii)</td><td></td><td>$\frac{480}{40} = 12$ or 12 to 1</td><td>B1

[1]</td><td>ft from (i)</td><td>their 480: their 40 isw
accept unsimplified answers</td></tr>
<tr><td>6</td><td>(iii)</td><td></td><td>
<table>
<tr><th>Transactions</th><th>Authorised</th><th>Un-authorised</th><th>Total</th></tr>
<tr><td>Queried</td><td>2 445</td><td>55</td><td>2 500</td></tr>
<tr><td>Not queried</td><td>497 495</td><td>5</td><td>497 500</td></tr>
<tr><td>Total</td><td>499 940</td><td>60</td><td>500 000</td></tr>
</table>
$\frac{2445}{5} = 489$ or 489 to 1</td><td>M1

DM1

A1

[3]</td><td>ft from (i)

cao</td><td>NB they are not required to complete the table.

{2500or 5xtheir 500}-(their 60-5) [=their 2445]

their 2445 ft from (i) :5</td></tr>
</table>

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4754: Applications of Advanced Mathematics (C4)

General Comments

There were questions in this summer's paper that were accessible to all candidates and few very low scores were obtained. There were also sufficient questions to challenge the more able candidates and few fully correct scores were seen. Questions 5, 7 and 8 provided the greatest challenge.

The comprehension was well understood and good scores were obtained here by most candidates.

As usual, loss of marks often followed poor algebra including errors when using negative signs and the absence of appropriate brackets. When integrating, it is still disappointing to see so few candidates adding the constant and then showing its evaluation. In this case, in 8(v), candidates, on seeing the given answer, too often either ignored '+c' completely or just wrote c=0 without justification.

The standard of work was, in general, good and the presentation was of a satisfactory standard.

Questions that were based on familiar methods were answered better than those where candidates had to think more for themselves.

Comments on Individual Questions

Paper A

1 The partial fractions method was well known and many candidates scored the full five marks. Candidates seemed well prepared for this question. There were some arithmetic errors and a few only put a constant on the numerator of the quadratic factor. Although it was not penalised on this occasion, it was disappointing to see so many candidates who found A, B and C correctly could not then accurately assemble the final result.

2 This binomial expansion question proved to be highest scoring question in Paper A. Few used an index other than 1/3. It was pleasing to note that unlike on some previous papers, most candidates did attempt the validity. On this occasion the expansion was valid at the endpoints but we accepted both inequality and 'less than or equal to' for both marks.

3 The first four marks were usually obtained. Candidates seemed to be well prepared for the 'R-method' and the number achieving full marks has improved. The most common error was the use of degrees instead of radians.

The final part, when finding the greatest and least possible values, caused some confusion. Whilst there were many completely correct solutions some omitted the 1 and just gave $\pm\sqrt{13}$ and many others tried to find angles from say solving $\sqrt{13}\sin(\theta - 0.983) = -1$ for θ.

4 (i) Quite a number of candidates overlooked the requirement to find the coordinates of x and y and so unnecessarily lost two marks.

Most candidates understood the method for finding the gradient of the curve but there were errors in the differentiation. Good candidates scored well here. Common errors included, $y=\cos 2\theta$, $\mathrm{d}y/\mathrm{d}\theta=2\sin\theta$, or $-2\sin\theta$, or $-\sin 2\theta$, or $-1/2\sin 2\theta$. Some of the incorrect differentiation lead to apparently the correct answer but obtained from wrong working.

(ii) Those that started with the correct double angle formula were usually successful here although some candidates left both x and θ in the answer.

5 This was a harder trigonometry question than on recent papers and few candidates scored the full six marks.

When using the first method, many failed to use the correct trigonometrical identity. Then, a common error was to 'lose' one of the factors eg $\cot^2\theta-2\cot\theta=0$, $\cot^2\theta=2\cot\theta$, $\cot\theta=2$. Others obtained $\cot\theta=0$ but did not find the solutions 90º and -90º (considering the graph of $\cot\theta$ or using $\cot\theta=\cos\theta/\sin\theta$ may have helped), often 180º, 0º, and -180º being given.

Those that chose to express their equation in terms of $\sin\theta$ and $\cos\theta$ from the outset usually obtained $\sin^2\theta+2\sin\theta\cos\theta-1=0$. Much then depended upon them using Pythagoras to obtain $2\sin\theta\cos\theta-\cos^2\theta=0$ and factorising. Once again, $2\sin\theta=\cos\theta$ was often seen and the $\cos\theta=0$ was forgotten.

Candidates would be advised to factorise instead of cancelling.

6 Many candidates obtained full marks in this volume of revolution question. Common mistakes included;

for the cone
- using the incorrect value of h (often $h=3$), or the incorrect value of r
- incorrectly expanding $\int_2^3 (3-y)^2 dy = \int_2^3 9+y^2 dy$
- omitting a negative sign $\int (3-y)^2 dy = (3-y)^3/3$
- using the wrong limits
- finding the area of a triangle

for the curve
- using the wrong limits (often 0-1 or 0-3)
- using the wrong substitution for x^2 (often $x^2=y^2/4$)

Most candidates did add their two parts together. The presence of the cone formula proved not to be a sufficient hint to deter many candidates from integrating.

7 (i) Many candidates scored full marks when finding the angle between the vectors. The vectors were almost always correct. The most common error was, after having obtained the vector AB correctly at the start then using it as $\begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$ when calculating the angle. This was a surprisingly common error.

(ii) A variety of methods were used when verifying the equation of the plane. These were often successful. For those who substituted points (which was quite common) the most frequent error was to substitute fewer than three points. For those who tried to establish the result by finding the scalar product with two vectors in the plane, the most common errors were that either they only used one vector or that they failed to substitute a point in order to find d.

There were some confused candidates who tried to substitute the vectors in the equation for the plane or to use position vectors of the points when finding the scalar products. Some confused d with –d.

A few used the vector equation of the plane, often with success, especially when substituting the general point into the plane equation.

The normal vector was usually found and used but candidates did not often find its scalar product with $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ having misunderstood how to find the angle the given plane

(iii) makes with the horizontal plane.

In the first part there were some circular arguments where CD was assumed to find k and then k was used to find CD. Those who realised they were required to substitute the coordinates of D in the equation of the plane were usually successful.

A large number failed to understand how to show that ABDC was a trapezium. The majority attempted to find vector CD (or DC), but did not always state it explicitly, and then correctly found the ratio of CD to AB-often by finding the lengths and stating $\sqrt{5}:\sqrt{20}$. Unfortunately this was often cancelled to 1:4.

8 Some candidates found this question difficult even though they were lead through it in stages and some straightforward marks were available.

(i) If they differentiated V=1/3 x^3 they were usually successful here.

(ii) There were many correct solutions in this part but those who failed to include +c or failed to substitute x=10 at t=0 lost marks.

(iii) This was usually successful. The most common error was $\sqrt{}$ (100-100k) =$\sqrt{}$100 -$\sqrt{}$100k.

(iv) Few candidates realised that they needed to start with dV/dt =1-kx. Those that did were usually successful.

(v) The algebra at the start of this part was poor. Negative signs missing or fiddled were often seen. A common error was $\frac{1}{1-x}-x-1=\frac{1-(1-x)(x-1)}{1-x}=\frac{x^2}{1-x}$. For the integration, separation of the variables and $\int dt=\int\frac{1}{1-x}-x-1dx$ was needed. Many failed to work with both sides of the equation. This then needed to be integrated and the constant of integration included, together with its clear evaluation. Most omitted the constant merely giving the given answer. Unnecessary marks were lost.

(vi) An appreciation that 1/0 or ln(1/0) is undefined was needed. There were many correct solutions. Some incorrectly gave ln (1/0) as ln 0 or 0.

Paper B

The Comprehension

1 This was very successful except when the evaluation was not shown.

2 (i) The most common error was not starting with 10,000. 10P4 was one of the commonest alternative starting points. Others included 10!,10!x4! and 10C4. Another common error was to assume that there was the same number of consecutive digits in both ascending and descending orders when these were not the same. The most common mark was M0 M1A0 followed by M1M1A0. There were few completely correct answers.

(ii) Candidates were allowed to follow through their result from (i) and still obtain full marks. Although the method was usually correct, answers were not given to an appropriate degree of accuracy as required in the question.

3 This was almost always correct.

4 Most candidates showed the correct method, with the failure to divide by 80 the most common mistake. However, there were a large number that did not correct this to the desired integer answer. There were also some interesting numbers of days in a year including 265.

5 The majority of candidates were not successful here and many omitted this part. The most common correct answer was that 'An attack can happen without a breach of the card's security'.

6 Candidates scored well in this question.

The most common error in the first part was putting 80 rather than 60 in the middle of the bottom row (and 60 not 40 above it).

In the second part answers were allowed as follow through marks from part(i) as were the method marks in the third part. The most common incorrect answer in the third part being 2425:5 with ratio 485:1. There were some unusual values in the tables following unclear methods. These occasionally included decimals, fractions and negative numbers.

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0